Summer 2015

Optimal dispatch in Smart Power Grids with partially known deviation

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OPTIMAL DISPATCH IN SMART POWER GRIDS WITH PARTIALLY KNOWN DEVIATION

by

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A thesis submitted in partial fulfillment of the
requirements for the Master of Science
degree in Electrical and Computer Engineering
in the Graduate College of
The University of Iowa

August 2015

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CERTIFICATE OF APPROVAL

MASTER'S THESIS

This is to certify that the Master's thesis of

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has been approved by the Examining Committee for the thesis requirement for the Master of Science degree in Electrical and Computer Engineering at the August 2015 graduation.

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Soura Dasgupta, Thesis Supervisor

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To my loving parents, brother and my ever-motivating husband.

ABSTRACT

We consider the optimal economic dispatch of power generators in a smart electric grid for allocating power between generators to meet load requirements at a minimum total cost. The first algorithm presented, is a distributed algorithm for frequency control and optimal dispatch, where, each generator independently adjusts its power-frequency set-point to erase power imbalance and load fluctuations by using the aggregate power imbalance in the grid, observed by local measurements of the frequency deviation. We also present a second decentralized consensus based algorithm where, we assume each generator, in addition to the measured frequency deviation in the grid, has minimal information exchange with its neighbors. Existing algorithms assume that frequency deviation is proportional to the load imbalance. In practice this is seldom exactly correct. We assume in both cases, that the only thing known about this relationship is that it is an unknown, odd, strictly increasing function. By simulations and mathematical proof of convergence, we provide verification of the efficiency of the algorithm.

PUBLIC ABSTRACT

Power grid is an interconnected system of supplying electricity from the supplier to the consumer, consisting of electricity generating plant, high voltage transmission lines- to carry electricity from the generating plant to the load center, and distribution lines- to carry electricity from load centers to individual consumers. A lot of research is being pursued to develop technologies for improving the next generation of power grid called the Smart Power Grid. The Smart Power Grid will have sophisticated communication infrastructure to improve the efficiency of electricity generation using renewable energy sources like the sun, water, etc and also to inform consumers of their electricity usage pattern. Also, the electricity market is now divided into three sections- generation, transmission and distribution. Private companies are competing with each other to provide electricity at the most competitive market price. We have developed two algorithms to help generating companies achieve their goal of meeting the hourly electricity need of the consumers and to do so at a minimum total cost.

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CHAPTER 1
INTRODUCTION

In this thesis, we present two distributed algorithms for optimal economic dispatch of power generators in a smart grid. The goal is to meet specified power generation requirements and do so at minimum total cost. Both assume that each generator can measure the frequency deviation of the grid. The second additionally assumes that , there exists, a local internet that permits generators to exchange their marginal costs with their neighbors. Using such information the generators must autonomously adjust their power output to asymptotically erase the load balance at a minimum cost.

The second algorithm does so asymptotically. While the first algorithm asymptotically erases the load deficit, it may attain the minimum cost solution since a zero load deficit is by itself a stationary point. However, this stationary point is locally unstable unless the minimum cost requirement is also met. Thus, in practice, inevitable load fluctuations will eventually cause the minimum cost solution to be attained.

The major difference between this work and earlier work undertaken is in the fact that, previous work as in [32] and [33] assume that the frequency deviation is proportional to the power imbalance. Thus its measurement is tantamount to knowing the load imbalance to within a positive constant of proportionality. Such a proportional relationship is only approximate and assumes small load imbalances. In practice the precise relationship between the imbalance and frequency deviation is unknown. Thus in this thesis, we extend [33] and [32] by relaxing the assumption of proportionality. Instead, we assume that all that is known about the frequency deviation is that it is an unknown odd increasing function of the load imbalance.

In this introductory chapter, we present some background information to reflect on the motivation behind the work presented in this thesis.

1.1 Origins of Modern Power Systems

In early days, power systems were small and localized [3], [7], [6] . The first such complete system was The Pearl Street Station, designed by Thomas Edison, in New York City, launched in 1882. It was a large central power plant that connected a 100-volt generator that burned coal to power a few hundred lamps in the neighborhood. Thereafter, many similar self-contained, isolated systems were built in the following years. There were two major types of systems: the AC and DC grids. Thomas Edison was a proponent of direct current, in which the electrons flow in a complete circuit, from the generator, through wires and devices, and back to the generator. A strong adversary to Edisons direct current (DC) technology was George Westinghouse's alternating current (AC) technology which had acquired many of the patents by Nikola Tesla.

The D.C. system of power distribution had its share of flaws. The direct-current system generated and distributed electrical power at the same voltage level as used by the customer's utilities. This is due to Ohms law: I = V/R, where I = current, V = voltage, and R = resistance. By increasing voltage, resistance is increased. The more resistance exists, the more electricity is lost as heat. Also, the higher the voltage, the smaller the wire can be used. As a result, DC generators had to be located within a mile of the load. This required the use of large, costly distribution wires and forced generating plants to be near the loads. On being transmitted over long distances, the signal strength diminished over long distances. AC had an obvious advantage over DC for the long distance transportation of electricity. AC generators could be constructed much further away as it was much easier and cheaper to step-up or step-down AC voltage using electric relays, invented and pioneered by Joseph Henry in the early 1830's. He envisioned that to ”step-up” the electrical signal in order to maintain signal strength, finer coil windings were needed to be employed over that of the thicker and costlier standard electrical telegraph sounder windings being utilized at the time. This modification of using the finer coil windings would cause the mechanism to ”trip” a relay switch which in turn would ”step-up” the electrical signal strength and allow it to maintain the telegraph signal over longer distances. This concept, apparently was not totally workable for the actual transmission of D.C. electrical generation systems nearly a half century later, even with the advancement of a ”three-wire” DC system over that of a ”two-wire” system. All of this led to the conclusion that A.C. electrical generating stations could be larger and cheaper to operate.

There were many technical changes in later Edison central power plants. Although he continued to support the use of direct-current (D.C.) for some years, when the new General Electric Company was formed from the Edison Electric Illuminating Company and others, it quickly adopted the more efficient alternating-current (A.C.) technology. However, the basic system of large central stations distributing power over a broader area remained, as did the basic uses for electricity, especially the familiar light bulb.

1.2 The Electric Grid

The power grid is an inter-connected network that transports the generated electricity from supplier and delivers it to the consumer [5]. It comprises of power plants generating electric power, high-voltage transmission lines to carry power from distant sources to load centers, and distribution lines that connect individual consumers.The traditional grid is designed based on the producer-controlled model with uni-directional power flow [13]. It constitutes of large,centralized power plants feeding power over an analog and electro-mechanical grid transmitting power to domestic as well as commercial users through high voltage transmission lines and substations. Such a set-up does not allow allow 2-way interaction between consumers and the grid. The present grid system suffers limited bandwidths, slower data transmission rates and there is no visibility in the distribution network below the substation.

Power stations may be classified as fossil fuel power stations, nuclear power-plants or can be fuelled by renewable energy sources such as hydel, solar [29] or geothermal plants. The conventional electric power is stepped up to a higher voltage before delivering it to the transmission network.

A transmission network is a large synchronous grid,that connects a large number of generators producing AC power, transports it along a long distance and delivers the power to the distribution sub-stations.

The final stage in the delivery of electric power is the distribution system. Once the power arrives at the distribution substation, it is stepped down to a medium voltage level and then is transported to the distribution lines. Primary distribution lines carry this power to distribution transformers which again lower the voltage to the appropriate customer voltage level.

Electrical grids systems can be organized in the following way [4]:

1. Radial set-up: It consists of networks organized as radial trees that begin with a power source and distribute electricity through networks with progressively lower voltages, eventually ending with consumers. 2. Mesh set-up: It is a radial structure but includes some additional lines which work as backups for rerouting power in the event of failure to a main line. 3. Looped (or Parallel path flow) set-up: In this system, several grid networks are interconnected so as to allow networks to share and balance the flow of electricity as required. In such a set-up, there can be some issues with control of power at network contact points.

1.3 Challenges of the Traditional Electric Grid

1. Limited delivery system: The traditional electricity grid uses a supervisory control and data acquisition system (SCADA) for power distribution which suffers limited bandwidths, slow data transmission rates and there is no visibility in the distribution network below the substation.

2. Inefficiency at managing peak load: Electricity demands vary through out the day, and so the cost to meet these changes as well. In the traditional grid, supply needs to change instantly according to the changing power demands and the power grid also needs to maintain a buffer of excess supply to meet increased demand. The ever increasing demand of electricity is leading to increased number of power stations. Peaking power

generators are used during peak hours for a short period each day. This results in higher cost of generation, lower efficiency and higher emissions.

3. More challenges to maintain stable power supply with the entry of alternative power generation sources: With its infinite availability and no harmful environmental effects, renewable energy resources have emerged as a potential energy resource for generation of electricity. But these energy resources do not produce un-interrupted power. Rather, the energy produced is intermittent and non-dispatchable during the times when they are not available, e.g., solar energy [24] at night, hydel energy during dry seasons. During such times, we need to have a backup network of traditional fuel power plants. Although, renewable energy generation represents a small portion of the total genearation, their energy capacity has grown rapidly in recent years. Therefore, the use of renewable energy promotes energy storage as well as advanced control techniques for grid stations to deal with this intermittency [39].

The above mentioned challenges and limitations of traditional electric grids calls for a more controlled and robust energy grid that is less dependent on centralized power stations, is bi-directional and more responsive to changing power demands [16, 27].

1.4 Modern Trends- The Smart Electric Grid

The drawbacks and limitations of the existing traditional grids are supposed to be alleviated by coming-of-age next generation Smart power grids [27]. The smart grid [9] is a modernized electrical grid that uses digital processing and communications technology to gather and act on information about the behaviours of suppliers and consumers in

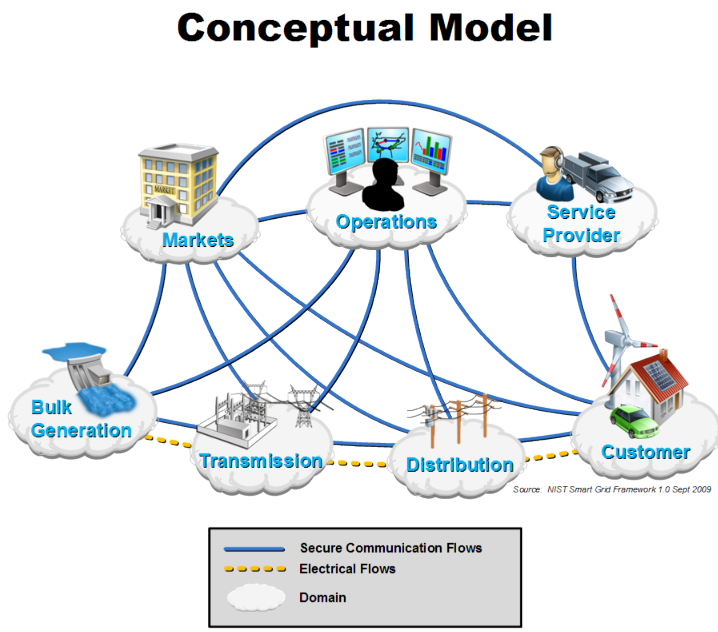

Figure 1.1: A model representation of smart grid, pic credits: [12]

an automated fashion to improve the efficiency, reliability, economics, and stainability of the production and distribution of electricity. IEEE defines smart grid as " an automated, widely distributed energy delivery network characterized by a two-way flow of electricity and information, capable of monitoring and responding to changes in everything from power plants to customer preferences to individual appliances". Electronic power conditioning and control of the production and distribution of electricity are important aspects of the smart grid.

According to the United States Department of Energy's Modern Grid Initiative report [8], [10] a modern smart grid must:

1. Be able to heal itself 2. Motivate consumers to actively participate in operations of the grid 3. Provide higher quality power that will save money wasted from outages 4.

Accommodate all generation and storage options 5. Enable electricity markets to flourish 6. Run more efficiently 7. Enable higher penetration of intermittent power generation

The main features of a smart grid are as follows [10]:

1. Enables active and informed consumer participation

Consumers have choices to motivate different purchasing patterns by using information about their electricity usage by smart metering technologies and by having information about electricity pricing and incentives. In this way, they can help balance supply and demand, and ensure electricity reliability.

2. Accommodates flexibity and reliability in network topology

A smart grid accommodates both large, centralised power plants and the growing number of customer-sited distributed energy resources. Integration of these resources including renewables and small-scale energy storage by allowing bi-directional energy flow, will increase the efficiency and reliability of the energy market. The smart grid will make use of technologies that improve fault detection and allow self-healing of the network without the intervention of technicians. This will ensure more reliable supply of electricity, and reduced vulnerability to natural or human-intervened disasters.

3. Facilitates load adjustment

The total load connected to the power grid can fluctuate significantly through out the day. The smart grid may warn all individual consumers or larger customers, to reduce the load temporarily. In a smart grid, the load reduction by even a small portion of the clients may eliminate the problem.

4. Provides demand response support

Varying degrees of automated communications allow generators and loads to interact in real time, coordinating demand to flatten spikes. Eliminating the fraction of demand that occurs in these spikes eliminates the cost of adding reserve generators, cuts wear and tear and extends the life of equipment, and allows users to cut their energy bills by telling low priority devices to use energy only when it is cheapest.

1.5 Three Levels of Control in Power system

If the total electric grid generation exceeds customer demand, frequency increases beyond the standard transmission level, until energy balance is achieved. Conversely, if there is a temporary generation deficiency, frequency declines until balance is again restored at a point below the scheduled frequency. The trend in system frequency is a measure of imbalance between load and generation. A certain amount of active power called frequency control reserve is available to correct the frequency of the system. Generally, Automatic Generation Control (AGC)has three levels of controls, used on multiple time scales to remove the power imbalance between load and generation [36].

Primary Control [1] is more commonly known as Frequency Response. Frequency Response takes place in a distributed fashion at individual generator level within the first few seconds following a change in system frequency to stabilize the grid. As frequency drops, motors will turn slower and draw lesser energy. Thus, each generating unit corrects the frequency deviation by increasing or decreasing the power generation. Rapid reduction of system load may also be effected by automatic operation of under-frequency relays which interrupt pre-defined loads within fractions of seconds or within seconds of frequency reaching a predetermined value. Primary Control protects the generator from the effects of excessively high frequency, but this especially corrects the frequency level more effectively when frequency level has dropped too low, i.e., generation loss causes sudden fall in grid frequency. The primary controller maintains the generation frequency by monitoring the amount of mechanical input to the shaft. The degree or slope of this is measured in percentage of the required frequency change to restore full generator capability against the frequency error. Primary Control does not return frequency to normal, but only stabilizes it. Other control components are used to restore frequency to normal.

Secondary control [1] is a centralized control scheme most commonly implemented through Load Frequency Control (LFC). LFC operates in conjunction with Supervisory Control and Data Acquisition (SCADA) systems for reducing the Area Control Error (ACE) to zero. ACE is the measure of imbalance between rated generation capacity and power consumption within the control area. It is measured with the knowledge of system frequency and net actual interchange, plus knowledge of net scheduled interchange.

$$ACE = \Delta(Net\ interchange) + (1/\beta)\Delta(f) \tag{1.5.1}$$

In the above equation, $\Delta(Net\ interchange)$ is the instantaneous difference between actual and scheduled interchange and $1/\beta$ is the frequency bias. The LFC algorithm adjusts the power generation levels in order to achieve power balance within the control area, involving only the generators in the local area. Secondary Control typically includes the balancing services deployed in the minutes time frame, typically every 5-15 minutes. Some resources however, such as hydroelectric generation, can respond faster in many cases.

Tertiary Control commonly known as the Economic Dispatch (ED) is also implemented from a centralized control station at the load serving entity (LSE) and transmission system operator (TSO). In existing grids, this control is implemented in a centralized fashion, and requires the central system to have the knowledge of all generators, making the control problem complex as the number of generators increases [36]. The ED process periodically re-allocates the total required power among generators to minimize total cost. The allocated power may deviate periodically because of cumulative load fluctuations and the actions of the secondary controller. The dispatch problem is typically formulated as a multivariable constrained optimization problem [19] that is then solved using Lagrangian techniques such as "lambda iteration" [37].

1.6 Economic Dispatch and Electricity Markets

Economic dispatch is defined by the EPAct section 1234 as:

The operation of generation facilities to produce energy at the lowest cost to reliably serve consumers, recognizing any operational limits of generation and transmission facilities. In the recent decades, the world-wide electricity markets have undergone deregulation, i.e., the electricity market has been divided and separated into three main segments-generation, transmission and distribution, thereby introducing market-based competition to the electric grid. Different energy suppliers and transmission companies compete on price to provide electricity to big and small utilities customers. Independent and private entities, acting as deregulated companies, employ dispatch algorithm to manage the generation schedule and then sell the generated energy to distributing firms that distribute this energy to the customer at market-wide competitive prices. Power producers that act as deregulated companies are assigned generation schedules according to the price competition of wholesale markets.

The economic dispatch has the following two fundamental components [2]:

1. Planning for tomorrows dispatch- involves scheduling available generating units for dispatch to cater to the electricity demand of each hour of the next days dispatch,based on forecast load for the next day, and doing so effectively and at optimal cost, taking into account, the variable operating costs. 2. Dispatching the Power System Today - involve monitoring of load and supply balance, system frequency and keeping transmission flows within safe limits.

1.7 Earlier work and our contribution

Traditionally, the dispatch problem is typically formulated as a centralized multivariate constrained optimization problem [19]. It is solved using Lagrangian techniques such as ”lambda iteration” [37] and complex numerical optimization methods such as genetic algorithms, particle swarm optimization or Monte-Carlo methods [18, 30] to determine the minimum cost allocation of power across generators. But in a smart grid, its expensive and unreliable to implement these centralized algorithms. The centralized controller requires an intensive connection and coordination among the power generators for collection of information, the failure of which may adversely affect the performance of the controller. Also a small change in the smart grid, with the variability and intermittence of alternative energy fuel supply, may lead to a total reconfiguration in the centralized algorithm which does not accommodate the intermittence characteristic of the smart grid .

Hence, a decentralized approach needs to be applied to the smart grid. We leverage recent advances in the distributed consensus theory to develop such control schemes. We believe that our broad approach extends well beyond the distributed dispatch problem to other control loops in the electric grid such as reactive power control and voltage regulation. Thirdly, the centralized controller is not able to accommodate the plug-and-play characteristic of smart grid. We use the terminology of the traditional economic dispatch in a broader sense than usual. Thus ”generators” represent all dispatchable units that have primary controllers that follow a power-frequency droop characteristic with negative slope, just like traditional generators. Though developed to ensure stable interconnection of synchronous generators [21], recent studies [22] have shown that the droop curve is useful and effective and it is advantageous to retain this mechanism even for modern microgrids. It is the droop curve that defines the aforementioned relationship between the load imbalance and the frequency deviation.

The dispatch algorithms discussed in this thesis have the following attractive and practical features: 1. Scalability: The distributed nature of our algorithm makes it more scalable, as opposed to centralized algorithms, and thus is a good choice for power grids employing large number of small distributed generators.

2. Dynamic Adaptability: The Distributed Algorithm automatically mitigates load imbalances by adjusting power allocations at optimum costs and hence this model is useful with highly variable loads and large number of intermittent alternative energy [25] genera-

tors in the grid.

3. Model Independence: The distributed algorithm does not require a detailed modeling of power flows as it handles the optimization problem iteratively.

The remainder of the thesis is organized as follows:

In Chapter 2, we analyze the properties of a distributed algorithm for optimal economic dispatch of power generators, for a non-linear power-frequency relation in the grid. Chapter 3 presents a new approach based on the distributed consensus theory for optimal economic dispatch. We also present simulations and results obtained by implementing this approach. Finally chapter 4 highlights the conclusion derived from our work, its future scope of extension and application of our work to the future smart grid.

CHAPTER 2
DISTRIBUTED ECONOMIC DISPATCH OF A NETWORK OF HETEROGENEOUS POWER GENERATORS

Earlier work in [33] presented a simple, distributed algorithm for frequency control and optimal economic dispatch of power generators, where each generator independently adjusts its power-frequency set-points of generators to correct for generation and load fluctuations using only the aggregate power imbalance in the network, which can be observed by each generator through local measurements of the frequency deviation on the grid. We try to analyze the properties of the algorithm in a practical environment where the power-frequency relations are not linear. We primarily intend to show that at the stationary point of the algorithm, i.e., when the power imbalance in the grid is erased to zero, marginal costs are equalized under practical constraints.

2.1 Model and Assumptions

We have the following Economic Dispatch model. Let us assume we have N generators that must supply the load power at any instant. At time-step k, we denote the total power consumed by $P_{load}[k]$ and the active power set point for generator i at the rated system frequency by $P_i(k),\ i \in 1 \ldots N$ The power imbalance in the system is thus given by:

$$\Delta P[k] = P_{load}[k] - \sum_{i=1}^{N} P_i[k] \tag{2.1.1}$$

We neglect the effects of reactive power flows, voltage deviations and transients.

We also do not consider power losses for simplicity. The actual active power produced by each generator is determined by its primary controller which uses $P_i(k)$ as a reference. More precisely, the primary controller on each generator responds to a power imbalance by adjusting its generated power relative to its generation set-point $P_i(k)$ until the imbalance is erased, i.e., by increasing the generated power above the active set-point in case of power deficit or decreasing it in case of power surplus at any given moment. In doing so, the primary controller implements a power-frequency characteristic (as in [26]) with a negative droop, so that a function of the active power $g(\Delta P[k])$ produced at time k is related to a function of the grid frequency $f[k]$ as:

$$\Delta f[k] = -g(\Delta P[k]) \tag{2.1.2}$$

The function $g(\cdot)$ conforms to the assumption below.

Assumption 2.1.1. The function $g(\Delta)$ is an analytic, strictly increasing and odd memoryless function in Δ. Further

$$\lim_{\Delta \to \infty} g(\Delta) = \infty.$$

This is analogous to the ACE observed by the secondary controller in a traditional LFC implementation. This is in marked contrast to earlier work in [32] and [33], that assume that for some possibly unknown positive constant β, $\Delta = -\beta \Delta f$

Let $J_i(P)$ be the cost function for generator i. The dispatch algorithm selects the $P_i[k]$ to force the power imbalance ΔP to zero and the marginal costs $J_i'(P_i)$ to eventually equalize. This in turn, minimizes the total cost $J = \sum_{i=1}^{N} J_i(P_i)$. The cost functions are assumed to be monotonically increasing, convex and bounded. The marginal costs are denoted as:

$$J_i'(P) \doteq \frac{dJ_i(P)}{dP}.$$

We make the following consumptions:

Assumption 2.1.2. $J_i(\cdot)$ is strictly convex and twice differentiable. Also, there exists $\eta_1 > 0$ and a finite positive nondecreasing function $f(\cdot)$, such that, the second derivative of the cost function

$$J_i''(P) \doteq \frac{d^2 J_i(P)}{dP^2},$$

satisfies $\eta_1 \leq J_i''(P) \leq f(P)$. Also, $J_i''(\cdot)$ is piece-wise continuous.

Assumption 2.1.3. There exists $\eta_2 > 0$, such that $J_i'(P) \geq \eta_2$. As the cost functions are convex, the marginal cost values, $J_i'(P_i)$ increase with increasing value of P_i.

The fact that the marginal costs are positive and increase with generation, accords with intuition and accounts for non-zero idling costs. Finally we denote

$$\mathbf{P} \doteq [P_1,\ P_2,\ \ldots,\ P_N]^T \tag{2.1.3}$$

i.e. $\mathbf{P}(\cdot) : \mathbb{R} \rightarrow \mathbb{R}^N$ has elements representing the power set-points across the generators.

2.2 Distributed Algorithm

We now revisit an algorithm, originally presented in [33].

The algorithm is an iterative algorithm. We note that in this context, since $g(\Delta P[k])$

is an increasing, odd function, this implies that a positive $g(\Delta P[k])$ ensures a positive $\Delta P[k]$ and a negative $g(\Delta P[k])$ ensures a negative $\Delta P[k]$ and vice-versa.

At time-step k, the updated generator power is given by:

$$P_i[k+1] = \begin{cases} P_i[k] + \left(\frac{\alpha_1 g(\Delta P[k])}{J_i'(P_i[k]) J_i''(P_i[k])}\right), & g(\Delta P[k]) \geq 0 \\ P_i[k] + \alpha_2 g(\Delta P[k]) \frac{J_i'(P_i[k])}{J_i''(P_i[k])}, & \text{else} \end{cases} \tag{2.2.4}$$

where α_1 and α_2 are positive rate controlling parameters and remain same for all generators.

From (2.2.4) we see that when the power imbalance $\Delta P[k]$ and hence $g(\Delta P[k])$ is positive, then the generators increase their rated powers inversely proportional to their marginal cost which implies that the generators with low marginal costs increase their allocation more rapidly than high cost generators. Conversely when the $\Delta P[k]$ and hence $g(\Delta P[k])$ is negative, then the low cost generator reduces its power less rapidly compared to high cost generators. The second derivative in the denominator implies that a large second derivative causes larger changes to the marginal costs.

This algorithm is implemented totally locally as the generator only needs to know its own cost function in addition to $g(\Delta P[k])$ which is the measured frequency deviation at any time. This algorithm tends to equalize the marginal costs across generators and thus leads to the minimum cost solution, when $P_{loss}[k] \equiv 0$.

2.3 Properties of Distributed Algorithm

As in [34], instead of analyzing the discrete time algorithm directly,we examine the continuous time version of the algorithm. This is so as under small enough gain α, the behaviour of one can be approximated by the behaviour of the other.

$$\frac{dP_i(t)}{dt} = \begin{cases} \alpha_1 g(\Delta P(t)) \left(\frac{1}{J_i'(P_i(t)) J_i''(P_i(t))} \right), & \text{if } g(\Delta P(t)) \geq 0 \\ \alpha_2(\Delta P(t)) \frac{J_i'(P_i(t))}{J_i''(P_i(t))}, & \text{otherwise} \end{cases} \tag{2.3.5}$$

Let $\mathbf{1} \doteq [1,\ 1, \ldots,\ 1]^T$ denote the "all-ones" vector. Then we can have:

$$\Delta P(t) \equiv P_{load}(t) - \mathbf{1}^T \mathbf{P}(t) \tag{2.3.6}$$

With a constant load, we have:

$$\Delta \dot{P}(t) = -\mathbf{1}^T (\mathbf{P}(t))$$

Consequently, with a positive $\Delta P(t)$ and hence a positive $g(\Delta P(t))$, the $\dot{P}_i$ are strictly positive, and

$$\begin{aligned} \Delta \dot{P}(t) &= -\mathbf{1}^T (\mathbf{P}(t)) \\ &\leq 0 \end{aligned} \tag{2.3.7}$$

Similarly, when $\Delta P(t)$ is negative, the $\dot{P}_i$ are non-positive, and

$$\begin{aligned} \Delta \dot{P}(t) &= -\mathbf{1}^T (\mathbf{P}(t)) \\ &\geq 0, \end{aligned} \tag{2.3.8}$$

with equality holding iff $g(\Delta P(t))$, or equivalently $\Delta P(t)$ equals zero. Thus in effect, we have proved the following theorem:

Theorem 2.3.1. Consider the algorithm in (2.3.5), with positive α_i, $\Delta P(t)$, defined in (2.3.6), $P_{load(t)}$ constant, and assumption 3.1.1 in force. Then,

$$\lim_{t\to\infty} \Delta P(t) = 0.$$

Thus, despite the fact that precise functional relationship between the load imbalance and the frequency deviation, the algorithm assuredly erases the load imbalance.

However, let us recall that our objective is not just to drive the imbalance to zero, but to converge to a point where the cost function is minimum as well. The difficulty is that $\Delta P(t) = 0$ is a stationary point of 2.3.5. Thus, the updates may cease before achieving the constrained optimum.

The rest of our analysis is devoted to demonstrating the following fact, that the only stationary point that is locally stable is one where $\Delta P(t) = 0$ and the marginals are all equal. As argued later, this is indeed the stationary point that we desire.

Now, from (2.3.5), we have:

$$\frac{d}{dt}\left\{J_i'(P_i(t))\right\}^2 = 2\alpha_1 g(\Delta P(t)). \tag{2.3.9}$$

Then, we have the following pivotal lemma.

Lemma 2.3.1. Under the conditions of Theorem 2.3.1, suppose $g(\Delta P(t)) \neq 0$, and for some i, j, $J_i'(P_i(t)) > J_j'(P_j(t))$. Then

$$\left(\frac{d(J_i'(P_i(t)) - J_j'(P_j(t)))}{dt}\right) < 0.$$

Proof. Let us first consider the case of $g(\Delta P(t)) > 0$. Then, we have the following:

$$\begin{aligned}
&\frac{d(J_i'(P_i(t)) - J_j'(P_j(t)))}{dt} \\
&= J_i''(P_i(t))\frac{dP_i(t)}{dt} - J_j''(Pj(t))\frac{dP_j(t)}{dt} \\
&= \alpha_1 g(\Delta P(t))\left(\frac{1}{J_i'(P_i(t))} - \frac{1}{J_j'(P_j(t))}\right) \\
&= \frac{\alpha_1 \Delta P(t)}{J_i'(P_i(t))J_j'(P_j(t))}\left(J_j'(P_j(t)) - J_i'(P_i(t))\right) \\
&< 0,
\end{aligned} \tag{2.3.10}$$

where the last inequality exploits the fact that the marginals are always positive.

On the other hand, when $g(\Delta P(t)) < 0$, we have:

$$\begin{aligned}
&\frac{d(J_i'(P_i(t)) - J_j'(P_j(t)))}{dt} \\
&= J_i''(P_i(t))\frac{dP_i(t)}{dt} - J_j''(P_j(t))\frac{dP_j(t)}{dt} \\
&= \alpha_2 g(\Delta P(t))\left(J_j'(P_j(t)) - J_i'(P_i(t))\right) \\
&< 0,
\end{aligned} \tag{2.3.11}$$

This lemma reveals a critical property of the algorithm: that it drives the marginal costs together, as long as a load imbalance persists. Of course this process of equalization ceases once a stationary point is attained. The next theorem shows that the only stationary point that is in fact locally stable is the *unique* point at which the marginals are equal and the load imbalance is zero.

Theorem 2.3.2. Suppose the conditions of Theorem 2.3.1 hold, and consider a stationary point, i.e., where $\Delta P = 0$, at which

(a) for some $\{i, j\} \subset \{1, \cdots, N\}$,

$$J_i'(P_i(t)) \neq J_j'(P_j(t)).$$

Then, this stationary point is unstable.

(b) for all $\{i, j\} \subset \{1, \cdots, N\}$,

$$J_i'(P_i(t)) = J_j'(P_j(t)).$$

Then this stationary point is unique and locally stable.

Proof. The convexity of the $J_i(\cdot)$ ensures that there is a one to one mapping between $J_i'(P_i)$ and P_i.

We first show that the stationary point in (b) is unique if it exists. To establish a contradiction suppose there are two such stationary points $\mathbf{P}_1$ and $\mathbf{P}_2$. Then at least one element of $\mathbf{P}_1$ is larger than its counterpart in $\mathbf{P}_2$, and another smaller. Then convexity precludes the possibility of equal marginal costs.

We define the Lyapunov function:

$$V(\mathbf{P}) = \sum_{i=1}^{N}\sum_{j=1}^{N}(J_i'(P_i) - J'(P_j))^2 + g((\Delta P(t)))^2.$$

We observe $V(\mathbf{P}) \geq 0$ with equality iff $\mathbf{P}$ is the unique stationary point defined in (b).

We first consider the case when $g(\Delta P(t)) \geq 0$. Then as $g(\cdot)$ is strictly increasing we have from 2.3.7 and theorem 2.3.2 :

$$\begin{aligned}
\dot{V}(\boldsymbol{P}) &= 2g(\Delta\boldsymbol{P}(t))g'(\Delta\boldsymbol{P}(t))\Delta\dot{\boldsymbol{P}}(t) + 2\sum_{i=1}^{N}\sum_{j=1}^{N}(J_i'(P_i(t)) - J_j'(P_j(t)))\frac{d}{dt}\left\{J_i'(P_i(t)) - J_j'(P_j(t))\right\} \\
&\leq 2g(\Delta\boldsymbol{P}(t))g'(\Delta\boldsymbol{P}(t))\Delta\dot{\boldsymbol{P}}(t) \\
&\leq 0,
\end{aligned}$$

Hence, $\dot{V}(\mathbf{P}) \leq 0$. The same thing can be shown when $g(\Delta P(t)) \leq 0$. Further, $\dot{V}$ is zero only at a stationary point. Thus, from [28], the stationary point in (b) is locally stable with equalization of marginal costs.

Now, consider a stationary point as in (a). At such a stationary point and its neighborhood, V is positive as at least one term in the summation is positive. Further, there exists an open set U with this stationary point on its boundary at which $V(\boldsymbol{P})$ is smaller. This follows readily from the first equation in the expression for $\dot{(V)}$. Thus as $\dot{V} \leq 0$, no trajectory in U, can return to this stationary point. Thus, indeed this stationary point is unstable.

Thus, as foreshadowed earlier, while the algorithm may momentarily converge to a stationary point at which the marginals are not equal, but the imbalance is zero, such a stationary point cannot be sustained.

Nonetheless eventual convergence to the optimum may be slow. This is not at all unexpected as the generation updates occur with the very little coordination among the generators. Accordingly, in the next chapter, we show that some modest additional coordination guarantees convergence to the global optimum.

CHAPTER 3
DISTRIBUTED CONSENSUS BASED ECONOMIC DISPATCH

Under this approach, as is the case cited in [32], there is a local internet that allows generators to independently make adjustments to their power-frequency primary controller set-points using the following three pieces of information: 1. their own marginal cost of generation; 2. the measured frequency deviation; 3. marginal generation cost of a subset of other generators

3.1 Problem Formulation

As in the problem considered in the last chapter,we have similar assumption of N generators that must supply the load power at any instant. We denote the load power by P_L which is taken to be constant in this work. Again, the active power set point for generator i at the rated system frequency is denoted by $P_i(k),\ i \in 1 \dots N$. The power imbalance in the system is thus given by:

$$\Delta(k) = P_L - \sum_{i=1}^{N} P_i(k) \tag{3.1.1}$$

We neglect the effects of reactive power flows, voltage deviations and transients. We also do not consider power losses for simplicity.

As discussed in the last chapter, the actual active power produced by each generator is determined by its primary controller which uses $P_i(k)$ as a reference, thereby, introducing a small frequency deviation $g(\Delta(k))$. Again, we assume that each controller measures

$g(\Delta(k))$, akin to the Area Control Error (ACE) signal observed by the secondary controller in a traditional Load Frequency Control (LFC) implementation [20], though it does not know its precise dependence on $\Delta(k)$. Instead all it knows is that $g(\cdot)$ conforms to the assumption below.

Assumption 3.1.1. The function $g(\Delta)$ is an analytic, strictly increasing and odd memoryless function in Δ. Further

$$\lim_{\Delta\to\infty} g(\Delta) = \infty.$$

Suppose $J_i(P_i)$ is the cost function for generator i. With $P = [P_1, \cdots, P_N]^\top$, we define P_i^* to be power allocations that minimize the total cost:

$$\sum_{i\in V} J_i(P_i) \tag{3.1.2}$$

$$\text{subject to} \sum_{i\in V} P_i = P_L \tag{3.1.3}$$

The goal of the dispatch algorithm is to choose the $P_i(k)$ so as to achieve:

$$\lim_{k\to\infty} P_i(k) = P_i^*. \tag{3.1.4}$$

The optimization (3.1.2,3.1.3) requires global communication between all generators. To circumvent this problem, [32] proposed an alternative cost function whose minimum coincides with the minimum of (3.1.2,3.1.3) but whose gradient descent minimization requires only local information exchange and the measurements the frequency deviation. However, [32] assumes that for some possibly unknown positive β, $g(\Delta) = \beta\Delta$. Thus effectively, it assumes that a quantity proportional to Δ is available. The knowledge of such a quantity, proves crucial to the generation of the gradient used in [32]. As in our work, we do not assume that $g(\Delta)$ can yield $\beta\Delta$, the algorithm of [32] cannot be implemented in the settings of this model.

3.2 A consensus Based Algorithm

As in [32], we assume that the network of generators and the communications infrastructure form a *possibly directed graph* $G = (V, E)$, where $V = \{1, \cdots, N\}$ is the vertex set indexing the generators. The directed edge $\{i, j\} \in E$ if generator i has access to generator j's marginal cost $J'_j(P_j)$. We define $\mathcal{N}(i)$ as the set of neighbors of i, i.e.

$$\mathcal{N}(i) = \{j \,|\, \{i, j\} \in E\}\,. \tag{3.2.5}$$

In the sequel we assume a constant load P_L, and a load deficit:

$$\Delta = P_L - \sum_{i=1}^{N} P_i. \tag{3.2.6}$$

We make the following assumption.

Assumption 3.2.1. The load P_L is constant. For all $i \in V$, the cost $J_i(\cdot) : \mathbb{R} \to \mathbb{R}$ is analytic everywhere. Further, there exists a $\gamma > 0$, such that for all $x \in \mathbb{R}$, and $i \in V$, there holds,

$$J''_i(x) \geq \gamma. \tag{3.2.7}$$

Finally, for every $i \in V$

$$\lim_{P_i \to \infty} J_i'(P_i) = \infty. \tag{3.2.8}$$

We note, (3.2.7) is a convexity assumption that is standard for most cost functions used in the power systems literature. Sometimes, these are obtained by interpolating tabulated data. These data are of a form that allows a convex interpolant, [15]. Convexity reflects the appealing reality that the marginal cost increases with production. We assume the marginal costs to be always positive, which is again a reality. For technical reasons we have not restricted the P_i to be nonnegative, though, in reality they would be.

Optimality of (3.1.3) subject to (3.1.2) necessitates that the marginal costs be equal subject to (3.1.2). Assumption 3.2.1 ensures that there is in fact a unique operating point meeting this requirement. To see this suppose a second operating point $\bar{P} \neq P^*$ has equal marginal costs and induces $\Delta = 0$. Call the i-th element of $\bar{P}$ and P^*, $\bar{P}_i$ and P_i^* respectively. Since $\Delta = 0$ in both cases

$$P_L = \sum_{i \in V} \bar{P}_i = \sum_{i \in V} P_i^*$$

and $\bar{P} \neq P^*$, there must be one element of $\bar{P}$ that is greater than the corresponding element of P^* and another that is less than the corresponding element of P^*. Thus for some i, $\bar{P}_i > P_i^*$ and for some j $\bar{P}_j < P_j^*$. As $J'(P_i^*) = J'(P_j^*)$, convexity ensures that $J'(\bar{P}_i) > J'(\bar{P}_j)$, establishing a contradiction.

Thus, the equality of the marginal costs subject to (3.1.2) is both necessary and sufficient for optimality. Thus, we must find P_i that equalize the marginals subject to

(3.1.2). The equalization of the marginals through their local exchange has similarities to the goals of consensus algorithms, [35]- [31]. An important difference is in the additional requirement of (3.1.2).

We define $P(k) = [P_1(k), \cdots, P_N(k)]^\top$. The algorithm we propose is as follows:

$$P(k+1) = P(k) - \mu z(k) \tag{3.2.9}$$

where μ is a suitably small adaptation gain and for some scalar $\alpha > 0$, the i-th element of $z(k)$ obeys

$$z_i(k) = -\alpha g(\Delta(k)) + J_i''(P_i) \sum_{j \in \mathcal{N}(i)} \left(J_i'(P_i) - J_j'(P_j) \right). \tag{3.2.10}$$

When $g(\Delta) = \beta\Delta$ then (3.2.10) is just the gradient of the cost function

$$S(P) = \frac{\alpha\Delta^2}{2\beta} + \frac{1}{2} \sum_{\{i,j\} \in E} \left(J_i'(P_i) - J_j'(P_j) \right)^2. \tag{3.2.11}$$

Indeed, [32] critically exploits this fact as under a connectedness assumption on G, the minimization of $S(P)$ is equivalent to the equalization of marginals and imbalance erasure. Of course $z(k)$ is no longer the gradient of $S(P)$ in our more complicated, albeit realistic, model for frequency deviation.

3.3 Stability Analysis

It is a well established fact in averaging theory, [17] that under suitably small $\mu > 0$ the asymptotic stability of the discrete time algorithm

$$x(k+1) = x(k) - \mu f(x(k))$$

can be concluded from the asymptotic stability of its continuous time counterpart:

$$\dot{x}(t) = -f(x(t)).$$

Thus, instead of (3.2.9, 3.2.10) we will analyze

$$\dot{P}_i = \alpha g(\Delta) - J_i''(P_i) \sum_{j \in \mathcal{N}(i)} \left(J_i'(P_i) - J_j'(P_j) \right) \tag{3.3.12}$$

We first prove the following.

Lemma 3.3.1. Consider (3.3.12) under assumptions 3.1.1 and 3.2.1, Δ is bounded from below.

Proof. Suppose $\Delta < 0$. Then $g(\Delta) < 0$. It suffices to show that in such a case P is bounded from above. Indeed without loss of generality assume at a given time t, $m \in V$ is such that

$$J_m'(P_m(t)) = \max_{i \in V} \{J_i'(P_i(t))\}.$$

Then as by Assumption 3.2.1, $J_m''(P_m(t)) > 0$, $\dot{P}_m < 0$. Consequently, P_m decreases in value. Thus by convexity at any given time, the largest marginal cost always declines. Thus the P_i are bounded.

Suppose now the lower bound on Δ is Δ_-. Define the cost function

$$J(P) = \alpha \int_{\Delta_-}^{\Delta} g(x)dx + \frac{1}{2} \sum_{\{i,j\} \in E} \left(J_i'(P_i) - J_j'(P_j) \right)^2 . \tag{3.3.13}$$

Clearly the integral in (3.3.13) is well defined and due to Assumption 3.1.1 nonnegative. Then we have the following Lemma.

Lemma 3.3.2. Suppose the graph G is connected. Then under the conditions of Lemma 3.3.1

$$\lim_{t \to \infty} \dot{P}(t) = 0. \tag{3.3.14}$$

Proof. Observe $J(P)$ is bounded from below. Further because of (3.1.1)

$$\begin{aligned}
\dot{J} &= \sum_{i=1}^{N} \frac{\partial J(P)}{\partial P_i} \dot{P}_i \\
&= \sum_{i=1}^{N} \alpha g(\Delta) \frac{\partial \Delta}{\partial P_i} \dot{P}_i \\
&+ \sum_{i=1}^{N} \left\{ J_i''(P_i) \sum_{j \in \mathcal{N}(i)} \left(J_i'(P_i) - J_j'(P_j) \right) \right\} \dot{P}_i \\
&= -\alpha g(\Delta) \sum_{i=1}^{N} \dot{P}_i \\
&+ \sum_{i=1}^{N} \left\{ J_i''(P_i) \sum_{j \in \mathcal{N}(i)} \left(J_i'(P_i) - J_j'(P_j) \right) \right\} \dot{P}_i \\
&= -\sum_{i=1}^{N} (\alpha g(\Delta) - J_i''(P_i) \sum_{j \in \mathcal{N}(i)} \left(J_i'(P_i) - J_j'(P_j) \right)) \\
&\quad \dot{P}_i \\
&= -\sum_{i=1}^{N} (\dot{P}_i)^2 \\
&= -\|\dot{P}\|^2 \\
&\leq 0. \qquad (3.3.15)
\end{aligned}$$

Thus $J(P)$ in (3.3.13) is bounded from above as well. As each summand in (3.3.13) is nonnegative, each must be bounded. The first ensures that $g(\Delta)$ and hence Δ is bounded. The second together with the connectedness of V and the convexity of the J_i ensures that $P_i - P_j$ is bounded for all $\{i, j\} \subset V$. Thus from (3.1.1) all P_i are bounded. Thus, as (3.3.12) has no explicit dependence on t, from Lassalle's Theorem, [28] P converges to the trajectory where $\dot{P} \equiv 0$.

We can now prove the main result.

Theorem 3.3.1. Under the conditions of Lemma 3.3.2 with P_i^* the values of P_i that optimize (3.1.2,3.1.3), (3.2.7) holds for all $i \in V$.

Proof. From Lemma 3.3.2 for all $i \in V$

$$\lim_{t \to \infty} \dot{P}_i(t) = 0.$$

Thus all variables including P_i and hence Δ have limit points. Consider two cases.

Case I: Limit point of Δ is nonnegative. Thus at this Δ, $g(\Delta) \geq 0$. Consider l so that $J_l'(P_l) \leq J_i'(P_i))$ for all $i \in V$. Then from (3.3.12) and Assumption 3.2.1 all summands in the expression of $\dot{P}_l$ are nonegative and must be zero. Thus in the limit $\Delta = 0$, for all $i \in \mathcal{N}(l)$, $J_l'(P_l) = J_i'(P_i))$ and for all $i \in V$

$$\sum_{j \in \mathcal{N}(i)} \left(J_i'(P_i) - J_j'(P_j) \right) = 0 \tag{3.3.16}$$

Then the marginal costs of all neighbors of elements of $\mathcal{N}(l)$ must also equal $J_l'(P_l)$. Continuing in this vein as G is connected all marginal costs are equal. As the point at which $\Delta = 0$ and the marginal costs are equal is unique, the result follows.

Case II: Limit point of Δ is nonpositive. The proof of this case is very similar to Case I. All that is needed is to choose l so that $J_l'(P_l) \geq J_i'(P_i))$ for all $i \in V$.

3.4 Simulations and Observations

In this section, we present simulations that demonstrate the performance of the approach described in this thesis.

We assume that $g(\triangle) = \triangle + \beta\triangle^3$. Fig. 3.2 shows the evolution of the power imbalance and total generation cost of a system with 6 generators. The cost curves and the total load for this simulation are the same as in the Example 2 in [14]. Specifically, the cost functions are of the form $J_i(P_i) = c_i P_i^2 + b_i P_i + a_i$, with the parameters $c_i,\ b_i,\ a_i$ as listed in Table 3.1.

Table 3.1: Parameter values for simulations.

Unit	1	2	3	4	5	6
a_i	1122	620	156	950	580.5	560.5
b_i	15.84	15.7	15.94	13.414	14.174	14.147
c_i	312E-5	388E-5	964E-5	264.1E-5	349.6E-5	349.6E-5

The generators are connected by a communication network represented by the undirected graph shown in Fig. 3.1. Observe that this is a connected graph that satisfies the assumptions of Theorem 3.3.1.

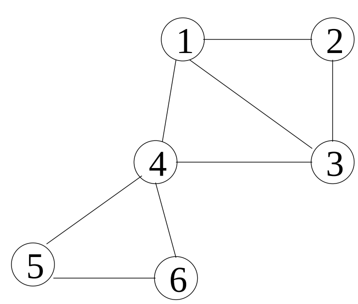

Figure 3.1: Connectivity graph for the simulated dispatch problem.

The cost functions are of the form $J_i(P_i) = c_i(P_i)^2 + b_i P_i + a_i$ with the parameters a, b and c as specified in the earlier paper. We initialized the simulation by setting all the six generators to generate equal power with a small randomly chosen initial power imbalance. We set $\alpha = 410^{-4}$, $\mu = 2$ and $\beta = 0.5$

When we initially start with the case where total generated power is greater than the load power, the power imbalance starts out as negative which is increased to zero and also during that time the total cost of generation steadily decreases within the first 500 iterations as shown in Figure 3.2 .

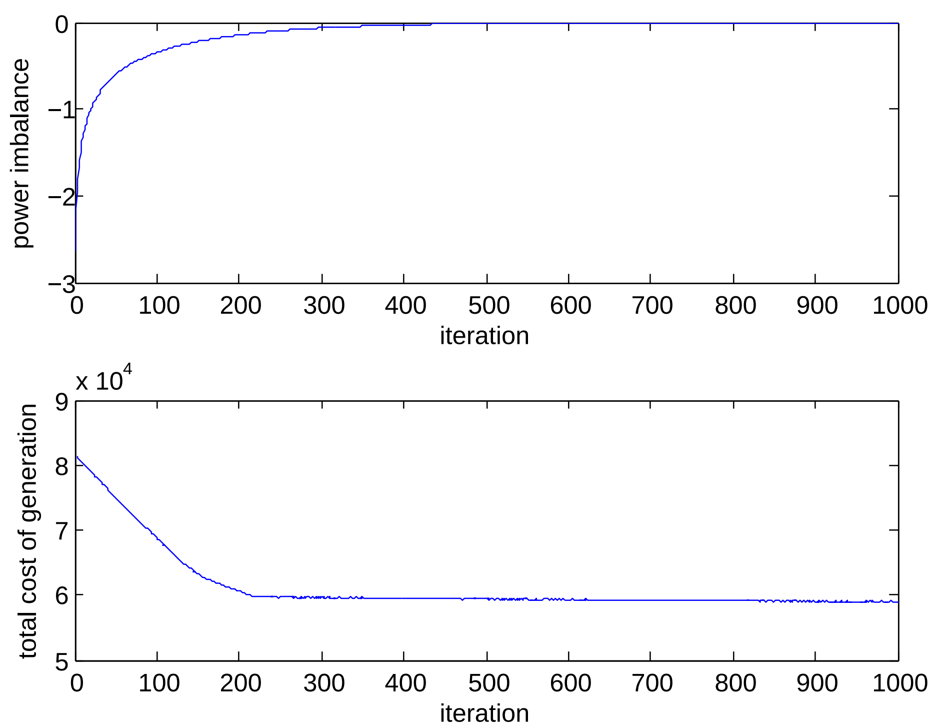

Figure 3.2: Connectivity graph for the simulated dispatch problem.

On the other hand, when we initially start with the case where total generated power is less than the load power, the power imbalance starts out as positive which is reduced to zero and also during that time the total cost of generation steadily decreases within the first 500 iterations as shown in Figure 3.3.

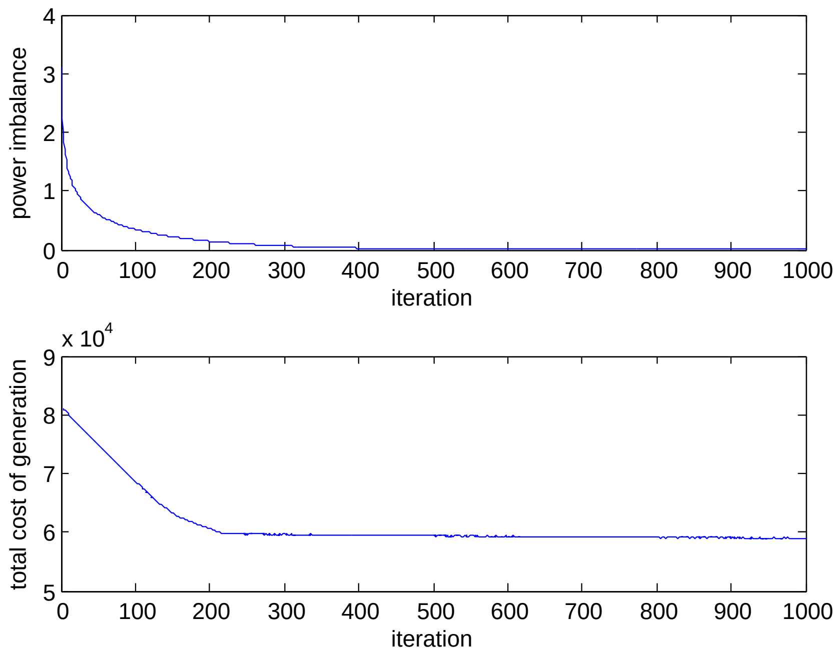

Figure 3.3: Connectivity graph for the simulated dispatch problem.

CHAPTER 4
ALGORITHM OPTIMIZATION AND FUTURE WORK

4.1 Conclusion

We have considered the optimal economic dispatch of power generators in a smart electric grid for allocating power between generators to meet load requirements at a minimum total cost. The algorithms presented are decentralized. Each generator independently adjusts its power output using only a measurement of the frequency deviation on the grid in the first algorithm and the second algorithm additionally involves minimal information exchange between generators. Existing algorithms assume that frequency deviation is proportional to the load imbalance. In practice this proportional relationship seldom holds. Accordingly our algorithms assume that the only thing known about this relationship is that it is an unknown, odd, strictly increasing function. We have shown that the distributed algorithm erases the load imbalance asymptotically with equalization of marginal costs, while the consensus based algorithm is globally convergent.

4.2 Future Scope and Algorithm Extension

An important future area of research is to tune this algorithm to grid dynamics to avoid instabilities, though it is safe to conjecture that sufficiently small μ and large enough sampling intervals in (3.2.9) should prevent grid instabilities. Our consensus based algorithm converges faster by using larger values of parameters but with the side-effects of increased instabilities. Hence, further work and investigation can be carried out to fix this problem and to balance out the pros and cons of the algorithm. We can also look into other forms of information exchanges that can further speed up convergence rates.

4.3 Application to Smart Grid

As stated earlier, the prime motivation behind the work in this thesis is the anticipated needs of next generation smart electric grids that will have smart consumer end-nodes and a high penetration of alternative energy generators. As alternative energy sources are intermittent in time and dispersed in geography, the electric grid must dynamically adjust generation and consumption. This stands in stark contrast to the traditional grid, where only a small number of large generation units are dispatchable. The future smart grid will likely have a plethora of small distributed generation (DG) [23], storage and demand-response units that will all contribute in varying measures. A centralized control approach will simply not scale and will lack the required agility. Thus the decentralized approach we have used in the work which needed a limited use of communication infrastructure and using local message exchanges will prove to be attractive and well suited to small grids with alternative energy generators at optimal market costs.

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